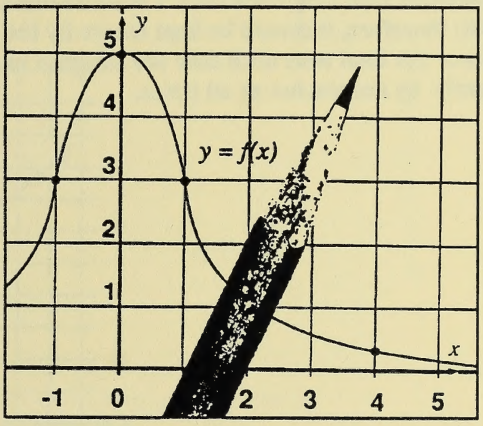
MATHEMATICS 33

Functions and Relations



Unit 3

Learning Facilitator's Manual

Distance Learning

Alberta EDUCATION

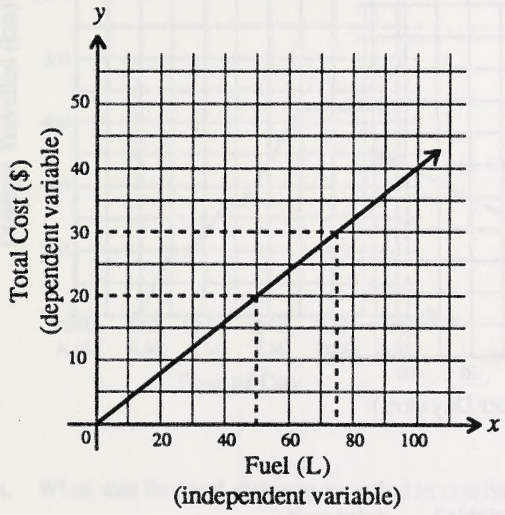
Note

This Mathematics Learning Facilitator's Manual contains answers to teacher-assessed assignments and the final test; therefore, it should be kept secure by the teacher. Students should not have access to these assignments or the final tests until they are assigned in a supervised situation. The answers should be stored securely by the teacher at all times.

Mathematics 33
Learning Facilitator's Manual
Unit 3
Functions and Relations
Alberta Distance Learning Centre
ISBN No. 0-7741-0141-5

Topic 1: Graphing Relations

1. The total cost to fill up a car with gasoline is related to the number of litres of fuel pumped into the tank at a certain price per litre. Draw the graph to demonstrate this relation for a price of 39.9¢/L. Label the graph correctly and indicate clearly the independent and dependent variables.



(3) a. From the graph determine the cost of 50 L of fuel.

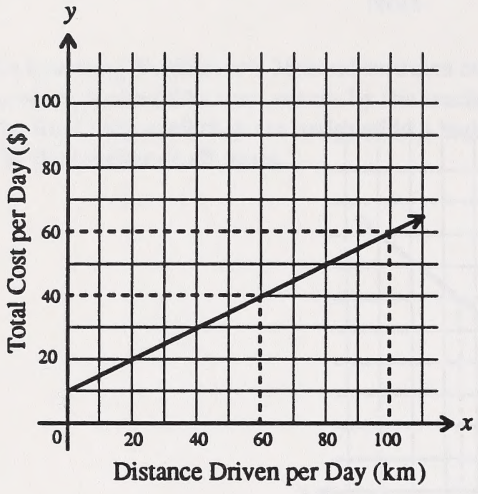
50 L of fuel will cost just under $20.00.

(3) b. From the graph determine the number of litres of fuel you could get for $30.00.

For $30.00 you would get about 75 L.

2. A car rental company rents a car for $10 per day plus $0.50 per kilometre driven.

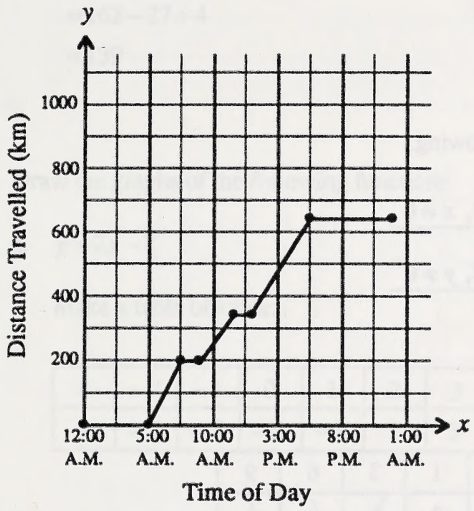
(2) a. Draw the graph of the cost per day versus the number of kilometres driven. Label the graph correctly.



(2) b. What is the dependent variable? **total cost**
What is the independent variable? **distance driven**

(2) c. From the graph determine the following:
- the total cost per day when 60 km are driven **$40**
- the total cost per day when 100 km are driven **$60**

3. The following graph represents one day in a family's trip by car from Edmonton to Jasper and then to Banff.



(2) a. What was the total distance travelled in one day? **650 km**

(2) b. What do the horizontal sections of the graph represent?
rest stops, lunch, sleep, etc.

(2) c. At what time did they leave Edmonton? **5:00 a.m.**

(2) d. At what time did they reach Banff? **5:30 p.m.**

Topic 1

_____ marks

Topic 2: Functions

(1) 1. Is the following relation a function? **yes**

$\{(12,1), (13,1), (14,20), (15,20)\}$

2. For the function $y = \frac{3}{x}$, do the following.

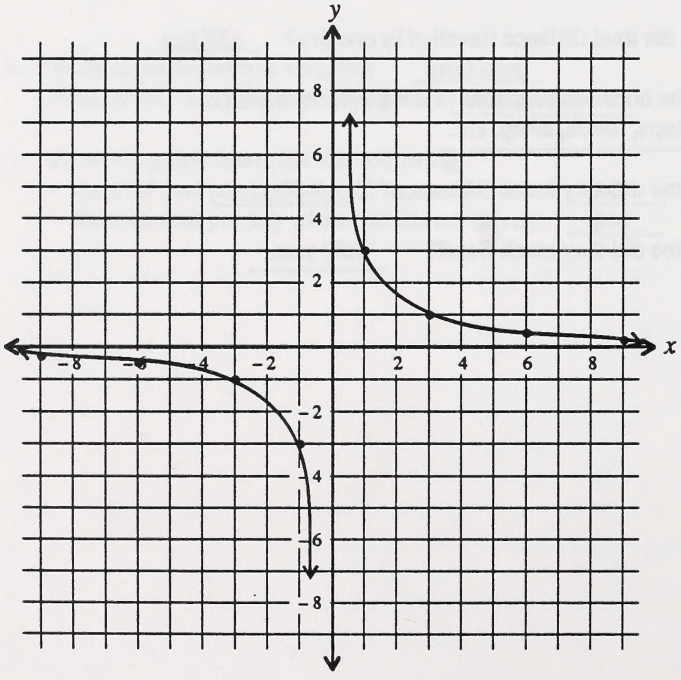
(1) a. State the domain. $x \in R, x \neq 0$

(1) b. State the range. $y \in R, y \neq 0$

(4) c. Draw the graph.

Make a table of values.

x	-9	-6	-3	-1	1	3	6	9
y	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	-3	3	1	$\frac{1}{2}$	$\frac{1}{3}$



(2) 3. Evaluate $f(x)=6x^3-9x+4$ for $f(3)$.

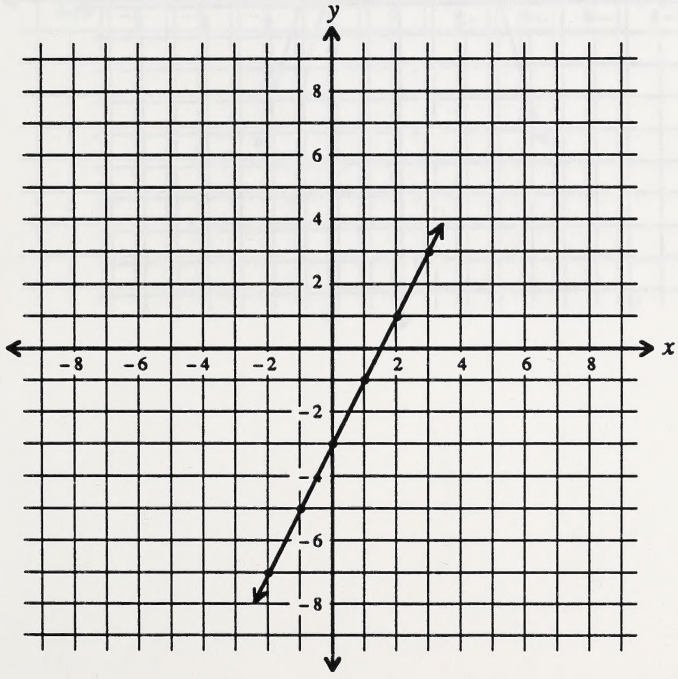
$$f(3)=6(3)^3-9(3)+4$$
$$=162-27+4$$
$$=139$$

4. Draw the graphs of the following functions.

(3) a. $y=2x-3$

Make a table of values.

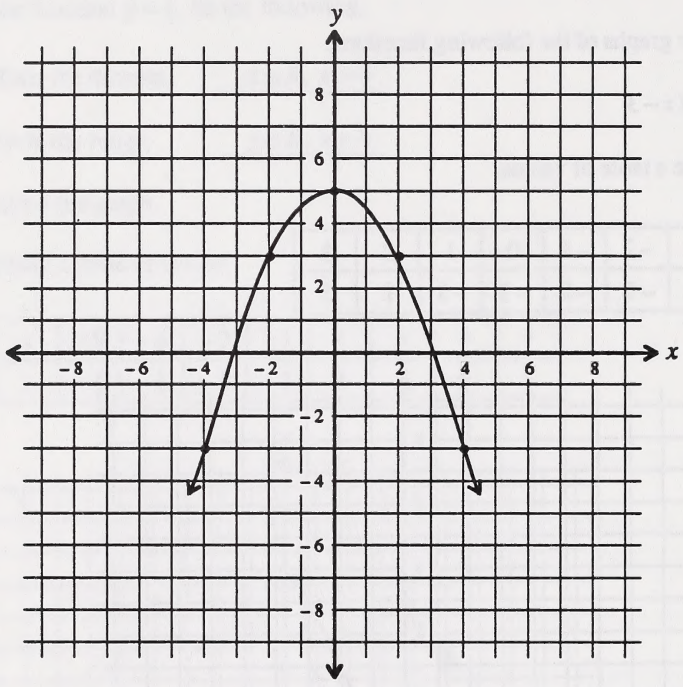
x	-2	-1	0	1	2	3
y	-7	-5	-3	-1	1	3



③ b. $y = -\frac{1}{2}x^2 + 5$

Make a table of values.

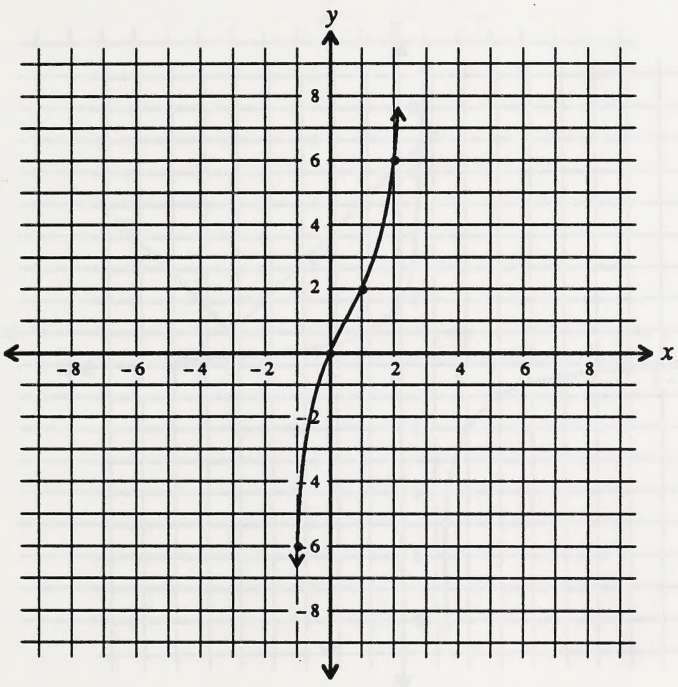
x	-4	-2	0	2	4	6
y	-3	3	5	3	-3	-13



③ c. $y = x^3 - 2x^2 + 3x$

Make a table of values.

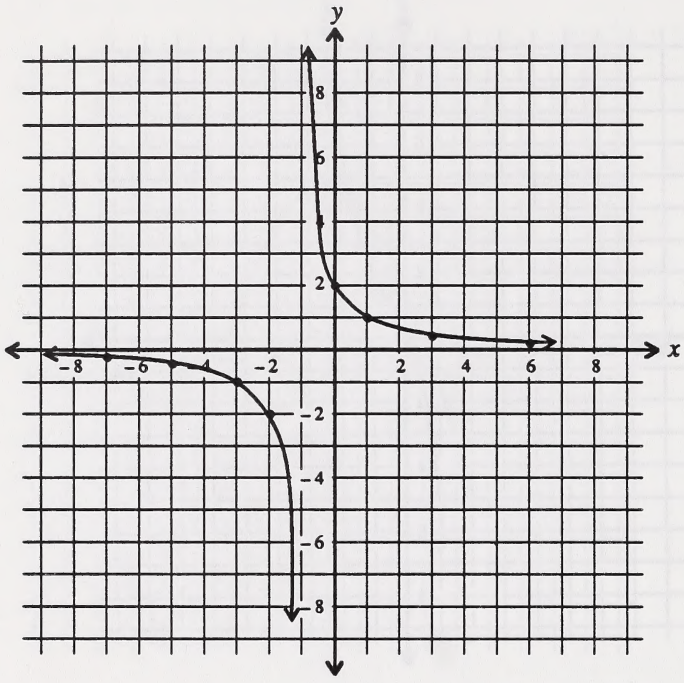
x	-2	-1	0	1	2	3
y	-22	-6	0	2	6	18



③ d. $y = \dfrac{2}{x+1}$

Make a table of values.

x	-7	-5	-3	-2	0	1	3	5
y	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	-2	2	1	$\frac{1}{2}$	$\frac{1}{3}$

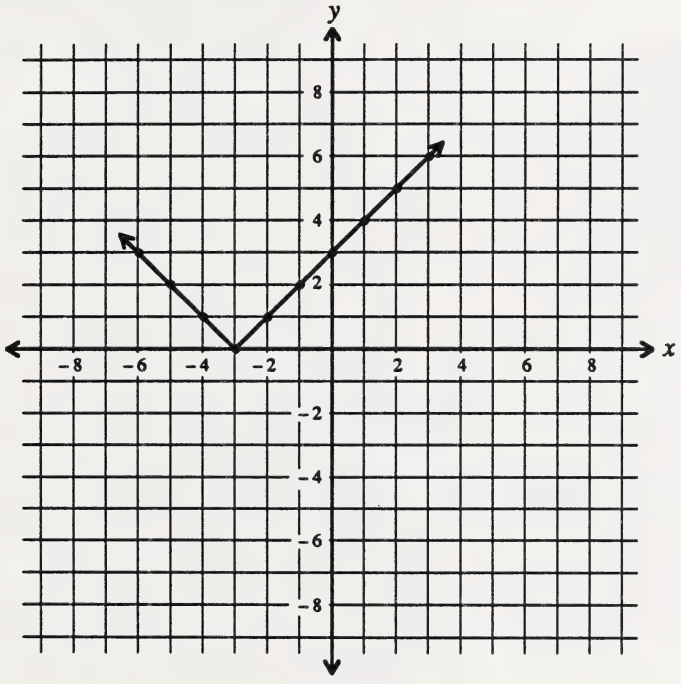


③

e. $y = |x+3|$

Make a table of values.

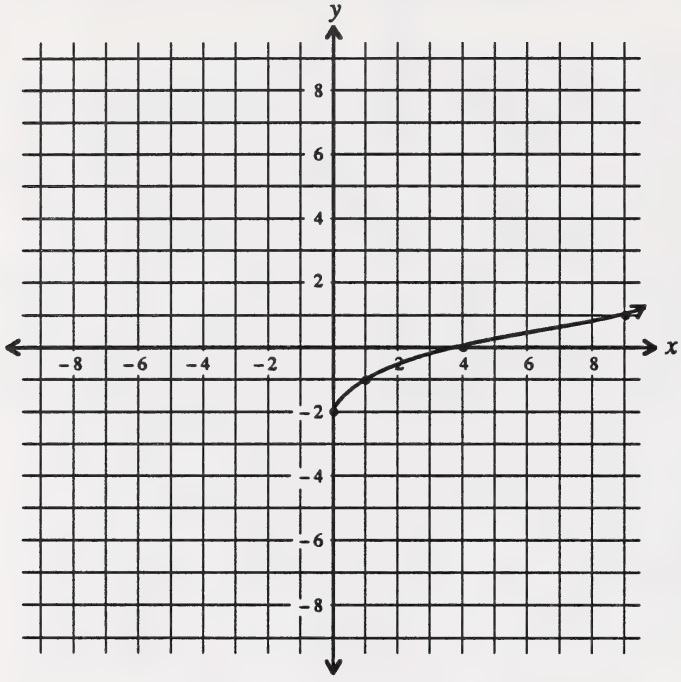
x	-6	-5	-4	-3	-2	-1	0	1	2	3
y	**3**	**2**	**1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**



③ f. $y = \sqrt{x} - 2$

Make a table of values.

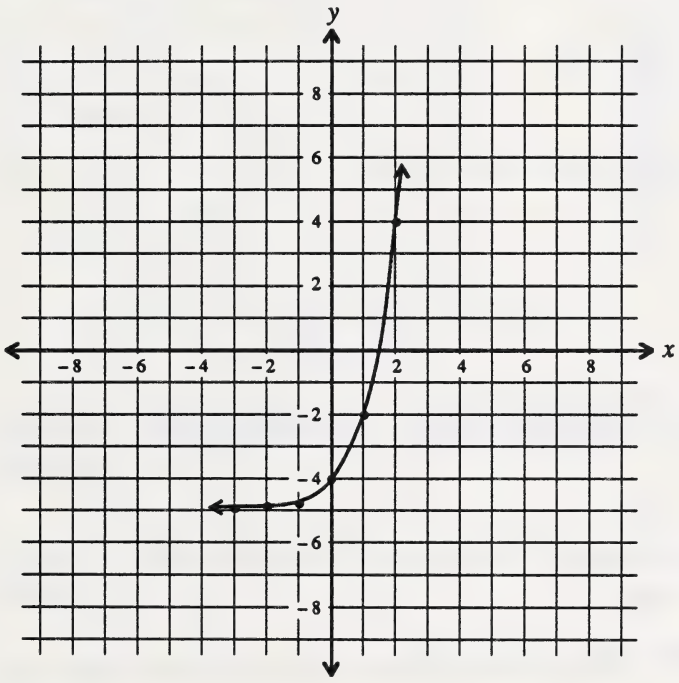
x	0	1	4	9
y	-2	-1	0	1



③ g. $y = 3^x - 5$

Make a table of values.

x	-3	-2	-1	0	1	2
y	$-4\frac{26}{27}$	$-4\frac{8}{9}$	$-4\frac{2}{3}$	-4	-2	4



5. The volume of water that can flow through a circular pipe is related to the diameter of the pipe. At a given pressure, this relation is written as

$V = kd^2$, where $k = 50 \text{ kL} / \text{cm}^2$,

$d =$ diameter of pipe in centimetres, and

$V =$ volume of water in kilolitres.

(2) a. What volume of water will flow through a pipe with a diameter of 2 cm?

$V = kd^2$

$V = 50 \times 2 \times 2$

$V = 200$

The volume of water is 200 kL.

(2) b. What volume of water will flow through a pipe with a diameter of 4 cm?

$V = kd^2$

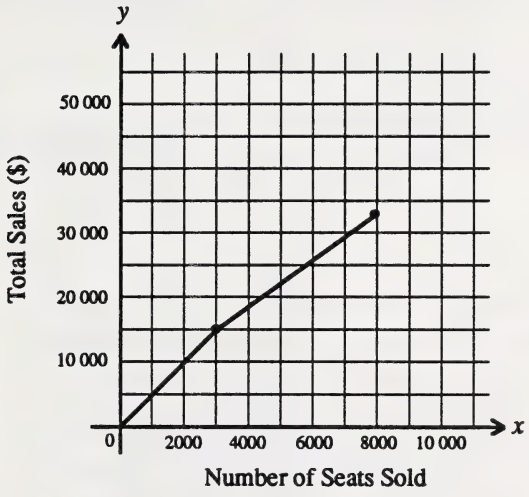
$V = 50 \times 4 \times 4$

$V = 800$

The volume of water is 800 kL.

6. There are 8000 seats in a local baseball stadium. Of the 8000, 3000 are gold seats at $5.00 each. The rest are blue seats at $3.50 each. All gold seats must be sold before blue seats can be sold.

(3) a. Draw the graph of the money raised for one game compared to the number of seats sold.



x	y
0	0
1000	5 000
2000	10 000
3000	15 000
4000	18 500
5000	15 500
6000	25 500
8000	32 500

(1) b. How many seats must be sold to cover the operating expenses of $20 000?
4429 seats

(1) c. What is the greatest possible **profit** (after operating expenses) from seat sales for one game?
$12 500

(1) d. How many seats must be sold to generate a total of $18 000?
3858 seats

Topic 2

_____ marks

Topic 3: Transformations of Functions

⑥ 1. Draw the graph of the function $y=|x|$. On the same grid draw the graph of the function $y=|x+8|-4$. Explain the transformation from $y=|x|$ to $y=|x+8|-4$.

$y=|x|$

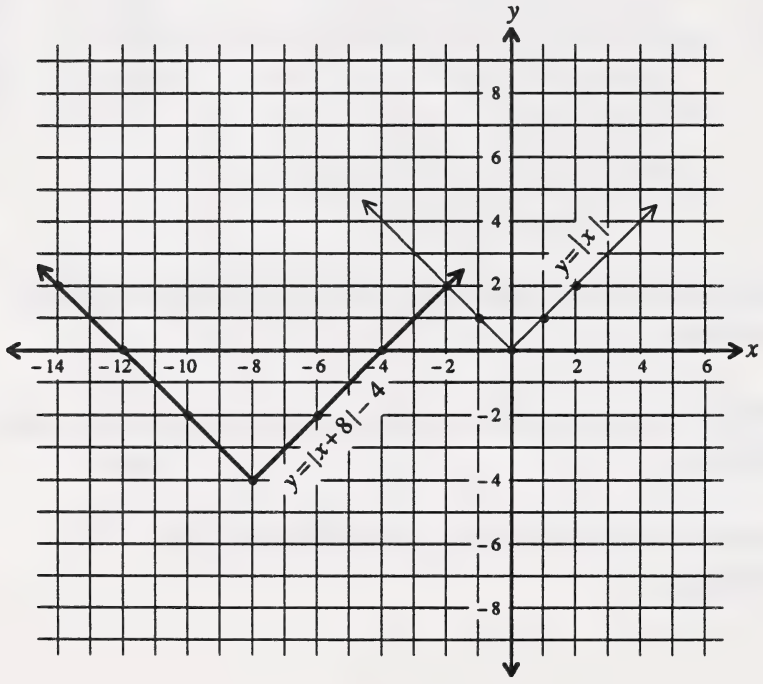
Make a table of values.

x	-2	-1	0	1	2	3
y	**2**	**1**	**0**	**1**	**2**	**3**

$y=|x+8|-4$

Make a table of values.

x	-14	-12	-10	-8	-6	-4	-2
y	**2**	**0**	**-2**	**-4**	**-2**	**0**	**2**



The graph of $y=|x+8|-4$ is eight units to the left and four units down from the graph of $y=|x|$.

⑥ 2. Draw the graph of the function $y = x^2$. On the same grid draw the graph of $y = -5x^2 + 4$. Explain the transformation effects.

$y = x^2$

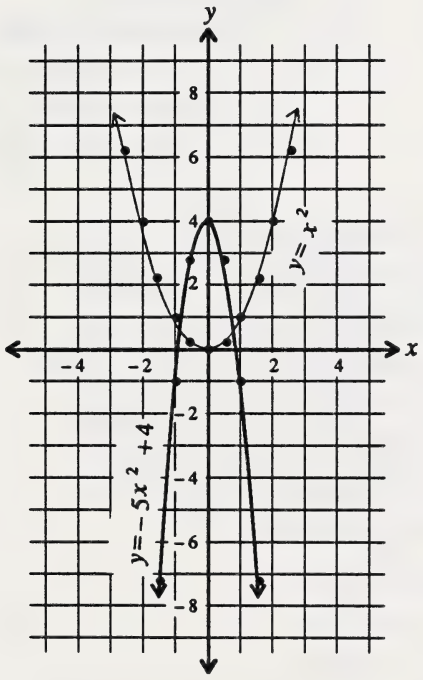
Make a table of values.

x	0	±0.5	±1	±1.5	±2	±2.5
y	0	0.25	1	2.25	4	6.25

$y = -5x^2 + 4$

Make a table of values.

x	0	±0.5	±1	±1.5	±2	±2.5
y	4	2.75	−1	−7.25	−16	−27.25



The negative sign causes the graph of $y = x^2$ to be reflected in the x-axis. The addition of 4 moves the graph up four units, and the 5 causes the new graph to be stretched vertically by a factor of 5.

(8)

3. The graph of $y = f(x)$ is shown. Draw the following graphs.

Make a table of values for $y = f(x)$.

x	-1	-2	-3	-4
y	1	1	2	1

a. $y = f(x)+4$
Make a table of values.

x	-1	-2	-3	-4
y	5	5	6	5

b. $y = f(x)-4$
Make a table of values.

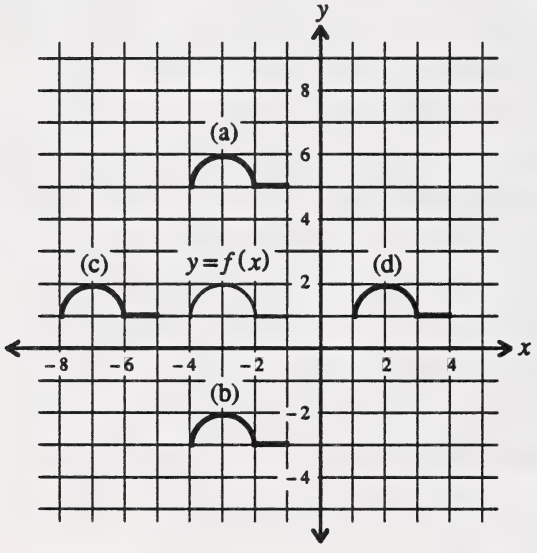
x	-1	-2	-3	-4
y	-3	-3	-2	-3

c. $y = f(x+4)$
Make a table of values.

x	-5	-6	-7	-8
y	1	1	2	1

d. $y = f(x-5)$
Make a table of values.

x	1	2	3	4
y	1	2	1	1



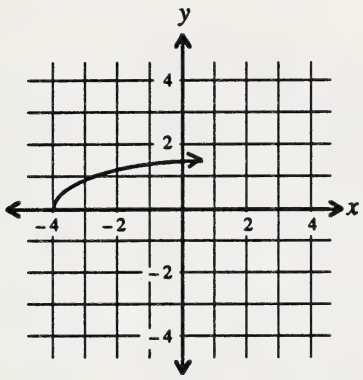
(4)

4. Write the new function that describes the transformation of $y = |x| + 2$ moved three units to the right and five units up.

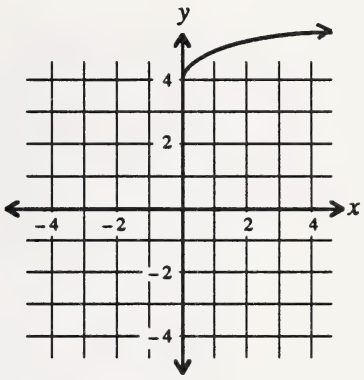
$y = |x-3| + 7$

④ 5. Which of the graphs best represents $y = \sqrt{x-4}$?

A.



B.



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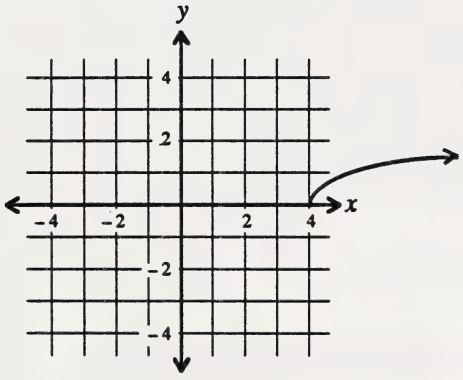
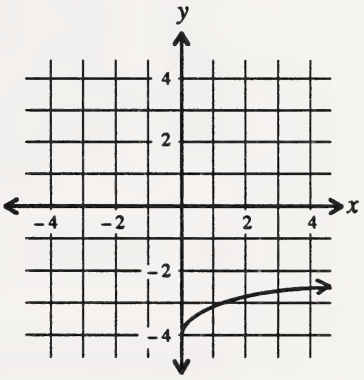
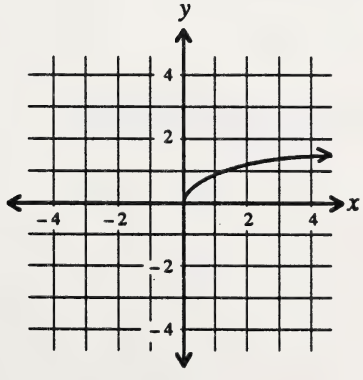


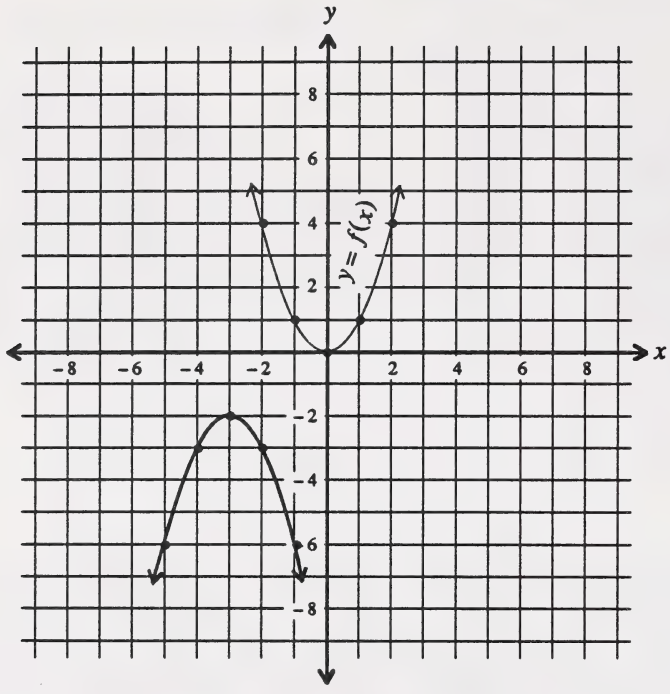
D.



E.



(4) 6. Write the new function that describes the transformation of $y = f(x)$ to the function shown.



The new function is $\underline{y = -f(x+3) - 2.}$

7. Write the new function that describes the following transformations.

(2) a. $y = (x-1)^3$ stretched vertically by a factor of 3

$y = 3(x-1)^3$

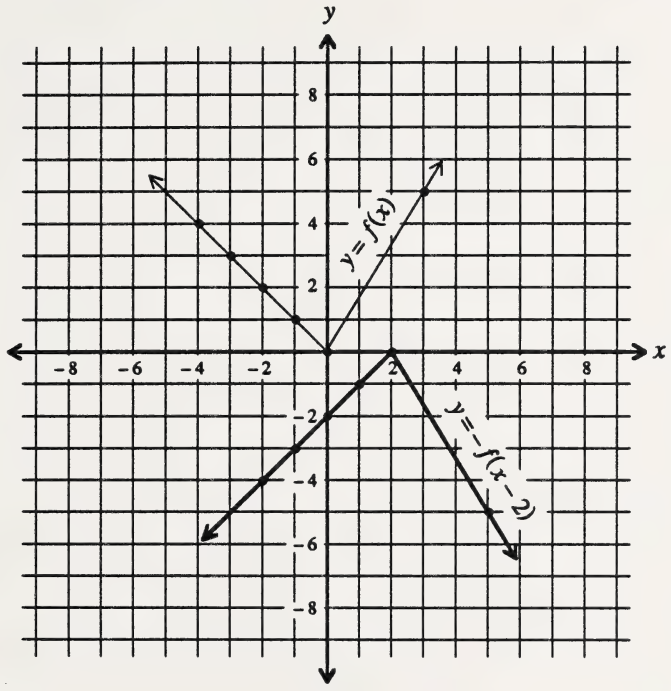
(2) b. $y = (x-1)^3$ compressed vertically by a factor of $\frac{1}{4}$ of the original

$y = \frac{1}{4}(x-1)^3$

(2) c. $y = (x-1)^3$ reflected in the x-axis

$y = -(x-1)^3$

(2) 8. Draw the graph of $y = -f(x-2)$.



Topic 3

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